Chapter 1

Parcialni derivace

V nasledujicich prikladech budeme hojne vyuzivat nasledujici vetu. Tato veta plyne z obdobne vety pro 1-dimenzionalni derivaci z 1. semestru. Neexistence v 2. casti vety neplyne z neexistence limity, ale z existence ruznych jednostrannych derivaci.

Veta 1.1. *Necht $n \in \mathbb{N}$, $i \in \{1, \ldots, n\}$, $a \in \mathbb{R}^n$ a $f : \mathbb{R}^n \to \mathbb{R}$. Necht e^i je i-ty kanonicky vektor a funkce $t :\to f(a + te^i)$ je spojita v 0. Pak plati:*

- *Jestlize existuje $\lim_{t\to 0} \frac{\partial f}{\partial x_i}(a + te^i) = A$, pak $\frac{\partial f}{\partial x_i}(a) = A$.*
- *Jestlize existuji $\lim_{t\to 0_+} \frac{\partial f}{\partial x_i}(a + te^i) = A$, $\lim_{t\to 0_-} \frac{\partial f}{\partial x_i}(a + te^i) = B$ a $A \neq B$, pak $\frac{\partial f}{\partial x_i}(a)$ neexistuje.*

Poznamka 1.2. *Necht $f(x, y) = f(y, x)$. Pak $\frac{\partial f}{\partial x}(x, y) = \frac{\partial f}{\partial y}(y, x)$, pokud alespon jedna z parcialnich derivaci existuje.*

Spojitost funkci v nasledujicich prikladech bude plynout z vet o spojitosti elementarnich funkci, polynomu a projekci, z vety o spojitosti a skladani funkci a spojitosti a aritmetickych operacich. Z techto vet plyne, ze pokud provedu operace skladani ci aritmeticke operace na funkce spojite na svych definicnich oborech, tak i vysledna funkce bude spojita na svem definicnim oboru. Rovnez vyuzivame, ze maximum dvou spojitych funkci je spojita funkce.

Priklad 1.3. *Pokud nebude uvedeno jinak, tak pro danou funkci f urcete definicni obor D_f, obor spojitosti C_f, rozhodnete v kterych bodech definicniho oboru existuji parcialni derivace a pokud existuji, tak je spoctete.*

1.

$$
\begin{aligned}
f(x, y) :=& x^m y^n : \ m, n \in \mathbb{N}, \\
D_f =& \mathbb{R}^2, \\
C_f =& D_f, \\
\frac{\partial f}{\partial x}(x, y) =& m x^{m-1} y^n && : [x, y] \in \mathbb{R}^2, \\
\frac{\partial f}{\partial y}(x, y) =& n x^m y^{n-1} && : [x, y] \in \mathbb{R}^2.
\end{aligned}
$$

2.

$$\begin{aligned} f(x,y,z) :=& xy + yz + zx, \\ D_f =& \mathbb{R}^3, \\ C_f =& D_f, \\ \frac{\partial f}{\partial x}(x,y,z) =& y + z && : [x,y,z] \in \mathbb{R}^3, \\ \frac{\partial f}{\partial y}(x,y,z) =& x + z && : [x,y,z] \in \mathbb{R}^3, \\ \frac{\partial f}{\partial z}(x,y,z) =& x + y && : [x,y,z] \in \mathbb{R}^3. \end{aligned}$$

3.

$$\begin{aligned} f(x,y) :=& e^{xy}, \\ D_f =& \mathbb{R}^2, \\ C_f =& D_f, \\ \frac{\partial f}{\partial x}(x,y) =& ye^{xy} && : [x,y] \in \mathbb{R}^2, \\ \frac{\partial f}{\partial y}(x,y) =& xe^{xy} && : [x,y] \in \mathbb{R}^2. \end{aligned}$$

4.

$$\begin{aligned} f(x,y) :=& x^y, \\ D_f =& \{[x,y] \in \mathbb{R}^2 : \ x > 0\}, \\ C_f =& D_f, \\ \frac{\partial f}{\partial x}(x,y) =& yx^{y-1} && : [x,y] \in D_f, \\ \frac{\partial f}{\partial y}(x,y) =& \log(x)x^y && : [x,y] \in D_f. \end{aligned}$$

5.

$$\begin{aligned} f(x,y,z) :=& \left(\frac{x}{y}\right)^z, \\ D_f =& \{[x,y,z] \in \mathbb{R}^3 : \ xy > 0\}, \\ C_f =& D_f, \\ \frac{\partial f}{\partial x}(x,y,z) =& \left(\frac{x}{y}\right)^z \frac{z}{x} && : [x,y,z] \in D_f, \\ \frac{\partial f}{\partial y}(x,y,z) =& -\left(\frac{x}{y}\right)^z \frac{z}{y} && : [x,y,z] \in D_f, \\ \frac{\partial f}{\partial z}(x,y,z) =& \left(\frac{x}{y}\right)^z \log\left(\frac{x}{y}\right) && : [x,y,z] \in D_f. \end{aligned}$$

6. *Domaci ukol:*

$$\begin{aligned} f(x,y,z) :=& x^{\frac{y}{z}}, \\ D_f =& \{[x,y,z] \in \mathbb{R}^3 : \ x > 0 \wedge z \neq 0\}, \\ C_f =& D_f, \\ \frac{\partial f}{\partial x}(x,y,z) =& \frac{y}{z} x^{\frac{y}{z}-1} && : [x,y,z] \in D_f, \\ \frac{\partial f}{\partial y}(x,y,z) =& \frac{\log(x)}{z} x^{\frac{y}{z}} && : [x,y,z] \in D_f, \\ \frac{\partial f}{\partial z}(x,y,z) =& -\frac{y\log(x)}{z^2} x^{\frac{y}{z}} && : [x,y,z] \in D_f. \end{aligned}$$

7. *Domaci ukol:*

$$\begin{aligned} f(x,y,z) :=& x^{y^z}, \\ D_f =& \{[x,y,z] \in \mathbb{R}^3 : \ x,y>0\}, \\ C_f =& D_f, \\ \frac{\partial f}{\partial x}(x,y,z) =& y^z x^{y^z-1} && : [x,y,z] \in D_f, \\ \frac{\partial f}{\partial y}(x,y,z) =& z y^{z-1} \log(x) x^{y^z} && : [x,y,z] \in D_f, \\ \frac{\partial f}{\partial z}(x,y,z) =& \log(x)\log(y) y^z x^{y^z} && : [x,y,z] \in D_f. \end{aligned}$$

8. *Pomoci retizkoveho pravidla spoctete* $\frac{\partial F}{\partial s}(s,t)$ *a* $\frac{\partial F}{\partial t}(s,t)$*, kde*

$$\begin{aligned} f(x,y,z) :=& xyz, \\ \varphi_1(s,t) :=& s+t^2, \\ \varphi_2(s,t) :=& st, \\ \varphi_3(s,t) :=& s, \\ F(s,t) :=& f(\varphi_1(s,t), \varphi_2(s,t), \varphi_3(s,t)). \\ \frac{\partial F}{\partial s}(s,t) =& 3s^2t + 2st^3, \\ \frac{\partial F}{\partial t}(s,t) =& s^3 + 3s^2t^2. \end{aligned}$$

9.

$$\begin{aligned} f(x,y) :=& e^{x|y|}, \\ D_f =& \mathbb{R}^2, \\ C_f =& D_f, \\ \frac{\partial f}{\partial x}(x,y) =& |y| e^{x|y|} : \ [x,y] \in \mathbb{R}^2, \\ \frac{\partial f}{\partial y}(x,y) =& \begin{cases} x \operatorname{sign}(y) e^{x|y|} & : y \neq 0, \\ 0 & : x = y = 0, \\ \textit{neexistuje} & : y = 0, x \neq 0, \end{cases} \\ \left(\frac{\partial f}{\partial y}\right)_{\pm}(x,0) =& \pm x. \end{aligned}$$

10.

$$\begin{aligned} f(x,y) :=& \sqrt{xy}, \\ D_f =& \{[x,y] \in \mathbb{R}^2 : \ xy \geq 0\}, \\ C_f =& D_f, \\ \frac{\partial f}{\partial x}(x,y) =& \begin{cases} \frac{y}{2\sqrt{xy}} & : xy > 0, \\ 0 & : y = 0, \\ \textit{neexistuje} & : x = 0, y \neq 0, \ f \textit{ neni definovana na okoli}, \end{cases} \\ \frac{\partial f}{\partial y}(x,y) :& \textit{ ze symetrie viz Poznamka } 1.2. \end{aligned}$$

11. *Domaci ukol:*

$$\begin{aligned}
f(x,y) :=& \sqrt{x^2+y^2},\\
D_f =& \mathbb{R}^2,\\
C_f =& D_f,\\
\frac{\partial f}{\partial x}(x,y) =& \begin{cases} \frac{x}{\sqrt{x^2+y^2}} & : [x,y] \in \mathbb{R}^2 \setminus \{[0,0]\},\\ \textit{neexistuje} & : x=y=0, \end{cases}\\
\left(\frac{\partial f}{\partial x}\right)_{\pm}(0,0) =& \pm 1,\\
\frac{\partial f}{\partial y}(x,y) :& \textit{ ze symetrie viz Poznamka } 1.2.
\end{aligned}$$

12.

$$\begin{aligned}
f(x,y) :=& \sqrt[3]{x^3+y^3},\\
D_f =& \mathbb{R}^2,\\
C_f =& D_f,\\
\frac{\partial f}{\partial x}(x,y) =& \begin{cases} \frac{x^2}{\sqrt[3]{(x^3+y^3)^2}} & : y \neq -x \ \left(x^3+y^3 = (x+y)\left(\frac{3}{4}x^2 + (\frac{x}{2}-y)^2\right)\right),\\ +\infty & : y=-x, x\neq 0,\\ 1 & : x=y=0, \end{cases}\\
\frac{\partial f}{\partial y}(x,y) :& \textit{ ze symetrie viz Poznamka } 1.2.
\end{aligned}$$

13.

$$\begin{aligned}
f(x,y) :=& |xy| \ (= |x||y|),\\
D_f =& \mathbb{R}^2,\\
C_f =& D_f,\\
\frac{\partial f}{\partial x}(x,y) =& \begin{cases} |y|\operatorname{sign}(x) & : x\neq 0,\\ 0 & : x=y=0,\\ \textit{neexistuje} & : x=0,\ y\neq 0, \end{cases}\\
\left(\frac{\partial f}{\partial x}\right)_{\pm}(0,y) =& \pm |y|,\\
\frac{\partial f}{\partial y}(x,y) :& \textit{ ze symetrie viz Poznamka } 1.2.
\end{aligned}$$

14. *Domaci ukol:*

$$\begin{aligned}
f(x,y) :=& \sqrt[3]{xy} \ (= \sqrt[3]{x}\,\sqrt[3]{y}),\\
D_f =& \mathbb{R}^2,\\
C_f =& D_f,\\
\frac{\partial f}{\partial x}(x,y) =& \begin{cases} \frac{1}{3}\sqrt[3]{\frac{y}{x^2}} & : x\neq 0,\\ 0 & : y=0,\\ +\infty\cdot y & : x=0, y\neq 0, \end{cases}\\
\frac{\partial f}{\partial y}(x,y) :& \textit{ ze symetrie viz Poznamka } 1.2.
\end{aligned}$$

15.

$$\begin{aligned}
f(x,y) :=& |y-x^2|,\\
D_f =& \mathbb{R}^2,\\
C_f =& D_f,\\
\frac{\partial f}{\partial x}(x,y) =& \begin{cases} -2x\,\mathrm{sign}(y-x^2) & : y \neq x^2,\\ 0 & : x=y=0,\\ \textit{neexistuje} & : y=x^2,\ x\neq 0,\end{cases}\\
\left(\frac{\partial f}{\partial x}\right)_{\pm}(x,x^2) =& \pm\, 2|x|,\\
\frac{\partial f}{\partial y}(x,y) =& \begin{cases} \mathrm{sign}(y-x^2) & : y \neq x^2,\\ \textit{neexistuje} & : y=x^2,\end{cases}\\
\left(\frac{\partial f}{\partial y}\right)_{\pm}(x,x^2) =& \pm\, 1.
\end{aligned}$$

16. *Domaci ukol:*

$$\begin{aligned}
f(x,y) :=& |y-x^3|,\\
D_f =& \mathbb{R}^2,\\
C_f =& D_f,\\
\frac{\partial f}{\partial x}(x,y) =& \begin{cases} -3x^2\,\mathrm{sign}(y-x^3) & : y \neq x^3,\\ 0 & : x=y=0,\\ \textit{neexistuje} & : y=x^3,\ x\neq 0,\end{cases}\\
\left(\frac{\partial f}{\partial x}\right)_{\pm}(x,x^3) =& \pm\, 3x^2,\\
\frac{\partial f}{\partial y}(x,y) =& \begin{cases} \mathrm{sign}(y-x^3) & : y \neq x^3,\\ \textit{neexistuje} & : y=x^3,\end{cases}\\
\left(\frac{\partial f}{\partial y}\right)_{\pm}(x,x^3) =& \pm\, 1.
\end{aligned}$$

17. *Domaci ukol:*

$$\begin{aligned}
f(x,y) :=& |y^2-x^2|,\\
D_f =& \mathbb{R}^2,\\
C_f =& D_f,\\
\frac{\partial f}{\partial x}(x,y) =& \begin{cases} -2x\,\mathrm{sign}(y^2-x^2) & : y \neq \pm x,\\ 0 & : x=y=0,\\ \textit{neexistuje} & : y=\pm x,\ x\neq 0,\end{cases}\\
\left(\frac{\partial f}{\partial x}\right)_{\pm}(x,x) =& \left(\frac{\partial f}{\partial x}\right)_{\pm}(x,-x) = \pm 2|x|,\\
\frac{\partial f}{\partial y}(x,y) :& \ \textit{ze symetrie viz Poznamka } 1.2.
\end{aligned}$$

18. *Domaci ukol:*

$$\begin{aligned}
f(x,y) :=& x\cdot[y] \ \text{([y] je cela cast y)},\\
D_f =& \mathbb{R}^2,\\
C_f =& \{[x,y]\in\mathbb{R}^2;\ x=0\vee y\notin\mathbb{Z}\},\\
\frac{\partial f}{\partial x}(x,y) =& [y]:\ \ [x,y]\in\mathbb{R}^2,\\
\frac{\partial f}{\partial y}(x,y) =& \begin{cases} 0 & : y\notin\mathbb{Z}\ (konstanta),\\ 0 & : x=0\ (konstanta),\\ neexistuje & : [x,y]\notin C_f,\end{cases}\\
\left(\frac{\partial f}{\partial y}\right)_+(x,n) =& 0:\ \ x\neq 0, n\in\mathbb{Z}\ (konstanta),\\
\left(\frac{\partial f}{\partial y}\right)_-(x,n) =& +\infty\cdot x:\ \ x\neq 0, n\in\mathbb{Z}.
\end{aligned}$$

19.

$$\begin{aligned}
f(x,y) :=& |y-\sin(x)|,\\
D_f =& \mathbb{R}^2,\\
C_f =& D_f,\\
\frac{\partial f}{\partial x}(x,y) =& \begin{cases} -\cos(x)\,\mathrm{sign}(y-\sin(x)) & : y\neq\sin(x),\\ 0 & : x=\frac{\pi}{2}+k\pi, y=(-1)^k,\ \ k\in\mathbb{Z},\\ neexistuje & : ostatni,\end{cases}\\
\left(\frac{\partial f}{\partial x}\right)_\pm(x,\sin(x)) =& \pm(-1)^k\cos(x):\ \ x\in\left(-\frac{\pi}{2}+k\pi,\frac{\pi}{2}+k\pi\right), k\in\mathbb{Z},\\
\frac{\partial f}{\partial y}(x,y) =& \begin{cases} \mathrm{sign}(y-\sin(x)) & : y\neq\sin(x),\\ neexistuje & : y=\sin(x),\end{cases}\\
\left(\frac{\partial f}{\partial y}\right)_\pm(x,\sin(x)) =& \pm 1.
\end{aligned}$$

20.

$$\begin{aligned}
f(x,y) :=& |\sin(y)-\sin(x)|,\\
D_f =& \mathbb{R}^2,\\
C_f =& D_f,\\
\frac{\partial f}{\partial x}(x,y) =& \begin{cases} -\cos(x)\,\mathrm{sign}(\sin(y)-\sin(x)) & : y\neq 2k\pi+x, y\neq(2k+1)\pi-x, k\in\mathbb{Z}\\ 0 & : x=\frac{\pi}{2}+k\pi, y=x+2l\pi,\ \ k,l\in\mathbb{Z},\\ neexistuje & : ostatni,\end{cases}\\
\left(\frac{\partial f}{\partial x}\right)_\pm(x,x+2l\pi) =& \pm(-1)^k\cos(x):\ \ x\in\left(-\frac{\pi}{2}+k\pi,\frac{\pi}{2}+k\pi\right), k,l\in\mathbb{Z},\\
\left(\frac{\partial f}{\partial x}\right)_\pm(x,(2l-1)\pi-x) =& \pm(-1)^k\cos(x):\ \ x\in\left(-\frac{\pi}{2}+k\pi,\frac{\pi}{2}+k\pi\right), k,l\in\mathbb{Z},\\
\frac{\partial f}{\partial y}(x,y) :&\ \text{ze symetrie viz Poznamka 1.2.}
\end{aligned}$$

21.

$$\begin{aligned} f(x,y) :=& \sqrt[3]{x+y^2},\\ D_f =& \mathbb{R}^2,\\ C_f =& D_f,\\ \frac{\partial f}{\partial x}(x,y) =& \begin{cases} \frac{1}{3\sqrt[3]{(x+y^2)^2}} & : x \neq -y^2,\\ +\infty & : x = -y^2, \end{cases}\\ \frac{\partial f}{\partial y}(x,y) =& \begin{cases} \frac{2y}{3\sqrt[3]{(x+y^2)^2}} & : x \neq -y^2,\\ +\infty \cdot y & : x = -y^2, y \neq 0,\\ \textit{neexistuje} & : x = y = 0, \end{cases}\\ \left(\frac{\partial f}{\partial y}\right)_{\pm}&(0,0) = \pm\,\infty. \end{aligned}$$

22.

$$\begin{aligned} f(x,y) :=& \begin{cases} \sqrt[3]{x^2+y}\log(x^2+y^2) & : [x,y] \neq [0,0],\\ 0 & : x = y = 0, \end{cases}\\ D_f =& \mathbb{R}^2,\\ C_f =& D_f. \end{aligned}$$

Spojitost v bode $[0,0]$ *plyne z nasledujicich odhadu a limity (predpokladame* $[x,y] \in B([0,0],1)$*):*

$$\begin{aligned} x^2+y^2 \geq& \frac{(|x|+|y|)^2}{2},\\ |\log(x^2+y^2)| \leq& \left|\log\left(\frac{(|x|+|y|)^2}{2}\right)\right|,\\ \left|\sqrt[3]{x^2+y}\right| \leq& \sqrt[3]{|x|+|y|},\\ |f(x,y)| \leq& \sqrt[3]{|x|+|y|}\left|\log\left(\frac{(|x|+|y|)^2}{2}\right)\right|,\\ \lim_{t\to 0}\sqrt[3]{t}\log\left(\frac{t^2}{2}\right) =& 0. \end{aligned}$$

$$\begin{aligned} \frac{\partial f}{\partial x}(x,y) =& \frac{2x\log(x^2+y^2)}{3\sqrt[3]{(x^2+y)^2}} + \frac{2x\sqrt[3]{x^2+y}}{x^2+y^2} : \ y \neq -x^2,\\ \left(\frac{\partial f}{\partial x}\right)_{\pm}(x,-x^2) =& \lim_{h\to 0\pm}\frac{\partial f}{\partial x}(x+h,-x^2)\\ =& \lim_{h\to 0\pm}\frac{2(x+h)\log((x+h)^2+x^4)}{3\sqrt[3]{((x+h)^2-x^2)^2}} + \frac{2(x+h)\sqrt[3]{(x+h)^2-x^2}}{(x+h)^2+x^4}. \end{aligned}$$

Pokud $x \neq 0$ *a* $x^2+x^4 \neq 1$ $(x \neq \pm\sqrt{\frac{\sqrt{5}-1}{2}})$*, tak prvni scitanec konverguje k*

$+\infty \cdot 2x \log(x^2+x^4)$. *Pokud* $x \neq 0$*, tak druhy scitanec konverguje k* 0*. Tedy*

$$\frac{\partial f}{\partial x}(x,-x^2) = +\infty \cdot 2x\log(x^2+x^4):\ x \notin \left\{0, \pm\sqrt{\frac{\sqrt{5}-1}{2}}\right\},$$

$$\left(\frac{\partial f}{\partial x}\right)_{\pm}(0,0) = \lim_{h\to 0\pm} 2\frac{\log(h^2)+3}{3\sqrt[3]{h}} = \mp\infty,$$

$$\frac{\partial f}{\partial x}\left(\pm\sqrt{\frac{\sqrt{5}-1}{2}}, \frac{1-\sqrt{5}}{2}\right) = 2x\lim_{h\to 0}\frac{\log(1+2xh+h^2)}{3\sqrt[3]{(2xh+h^2)^2}} = 0.$$

Pouzili jsme, ze $\lim_{t\to 0}\frac{\log(1+t)}{t} = 1$, $t = 2xh + h^2 \neq 0$*, kde* $h \in \left(-\frac{1}{2}, \frac{1}{2}\right) \setminus \{0\}$.

$$\frac{\partial f}{\partial y}(x,y) = \frac{\log(x^2+y^2)}{3\sqrt[3]{(x^2+y)^2}} + \frac{2y\sqrt[3]{x^2+y}}{x^2+y^2}:\ y \neq -x^2,$$

$$\left(\frac{\partial f}{\partial y}\right)_{\pm}(x,-x^2) = \lim_{h\to 0\pm}\frac{\partial f}{\partial x}(x, h-x^2)$$

$$= \lim_{h\to 0\pm}\frac{\log(x^2+(h-x^2)^2)}{3\sqrt[3]{h^2}} + \frac{2(h-x^2)\sqrt[3]{h}}{x^2+(h-x^2)^2}.$$

Pokud $x \neq 0$ *a* $x^2 + x^4 \neq 1$ *(*$x \neq \pm\sqrt{\frac{\sqrt{5}-1}{2}}$*), tak prvni scitanec konverguje k* $+\infty \cdot \log(x^2+x^4)$*. Pokud* $x \neq 0$*, tak druhy scitanec konverguje k* 0*. Tedy*

$$\frac{\partial f}{\partial y}(x,-x^2) = +\infty \cdot \log(x^2+x^4):\ x \notin \left\{0, \pm\sqrt{\frac{\sqrt{5}-1}{2}}\right\},$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{h\to 0}\frac{\log(h^2)+6}{3\sqrt[3]{h^2}} = -\infty,$$

$$\frac{\partial f}{\partial y}\left(\pm\sqrt{\frac{\sqrt{5}-1}{2}}, \frac{1-\sqrt{5}}{2}\right) = \lim_{h\to 0}\frac{\log(1-2x^2h+h^2)}{3\sqrt[3]{h^2}} = 0.$$

Pouzili jsme, ze $\lim_{t\to 0}\frac{\log(1+t)}{t} = 1$, $t = -2x^2h + h^2 \neq 0$*, kde* $h \in \left(-\frac{1}{2}, \frac{1}{2}\right) \setminus \{0\}$.

23.

$$f(x,y) := \begin{cases} e^{\frac{-1}{x^2+xy+y^2}} & : [x,y] \neq [0,0], \\ 0 & : x = y = 0, \end{cases}$$
$$D_f = \mathbb{R}^2,$$
$$C_f = D_f.$$

Pouzivame $x^2 + xy + y^2 = \frac{3}{4}x^2 + \left(\frac{1}{2}x + y\right)^2 > 0$ *pro* $[x,y] \neq [0,0]$*. Spojitost v bode* $[0,0]$ *plyne z vety o limite slozene funkce, predchoziho odhadu a* $\lim_{t\to-\infty} e^t = 0$.

$$\frac{\partial f}{\partial x}(x,y) = \frac{(2x+y)e^{\frac{-1}{x^2+xy+y^2}}}{(x^2+xy+y^2)^2}:\ [x,y] \neq [0,0],$$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0}\frac{\partial f}{\partial x}(h,0) = \lim_{h\to 0}\frac{2he^{\frac{-1}{h^2}}}{h^4} = 0.$$

Posledni rovnost bud zname a nebo odvodime pomoci l'Hopitala.

$$\frac{\partial f}{\partial y}(x,y): \quad \textit{ze symetrie viz Poznamka } 1.2.$$

24.

$$\begin{aligned}
f(x,y) :=& \max\{y-\cos(x),0\},\\
D_f =& \mathbb{R}^2,\\
C_f =& D_f,\\
\frac{\partial f}{\partial x}(x,y) =& \begin{cases} \sin(x) & : y > \cos(x),\\ 0 & : y < \cos(x) \vee (y=\cos(x), x \in \{k\pi;\ k\in\mathbb{Z}\}),\\ \textit{neexistuje} & : \textit{ostatni}, \end{cases}\\
\left(\frac{\partial f}{\partial x}\right)_+ (x,\cos(x)) =& \sin(x): \ x \in (2k\pi,(2k+1)\pi), k\in\mathbb{Z},\\
\left(\frac{\partial f}{\partial x}\right)_+ (x,\cos(x)) =& 0: \ x \in ((2k+1)\pi,(2k+2)\pi), k\in\mathbb{Z},\\
\left(\frac{\partial f}{\partial x}\right)_- (x,\cos(x)) =& 0: \ x \in (2k\pi,(2k+1)\pi), k\in\mathbb{Z},\\
\left(\frac{\partial f}{\partial x}\right)_- (x,\cos(x)) =& \sin(x): \ x \in ((2k+1)\pi,(2k+2)\pi), k\in\mathbb{Z},\\
\frac{\partial f}{\partial y}(x,y) =& \begin{cases} 1 & : y > \cos(x),\\ 0 & : y < \cos(x),\\ \textit{neexistuje} & : \textit{ostatni}, \end{cases}\\
\left(\frac{\partial f}{\partial y}\right)_+ (x,\cos(x)) =& 1,\\
\left(\frac{\partial f}{\partial y}\right)_- (x,\cos(x)) =& 0.
\end{aligned}$$